Can One Measure the Weak Phase of a Penguin Diagram?

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Thank you very much.

Particularly compelling about CP violation in B - All three angles can in principle be extracted <u>cleanly</u> i.e. without too much theoretical uncertainity

Hope is to test CKM and determine whether CP-violating New Physics (NP) exists beyond SM.

Obvious question?
How can NP affect CP asymmetries?

Two ways: NP can affect

1. B decays - only modes with decay process dominated by penguin diagram rather than tree affected.
2. B mixing - affect extraction of V_{ts}, V_{td}
   affect measurement of α, β

α measured by $B_d(t) \to \pi\pi$ or $\rho\pi$

$\alpha \to \alpha + \theta_{NP}$

β measured by $B_d(t) \to \psi K_s$

$\beta \to \beta - \theta_{NP}$

γ measured $B^{\pm} \to DK^{\pm}$, $D^* K^{*\pm}$

$\gamma \to \gamma$

θ_{NP} cancels in the sum $\alpha + \beta + \gamma$

New physics found if measurement of angles inconsistent with measurement of sides

- Allowed region of unitarity triangle still fairly large. Concievable that even in presence of NP △ constructed by α, β, γ still lies within the allowed range
- α, β, γ △ lies outside the allowed region – Is it new physics or underestim. theoretical uncertainity which constrains unitarity △

→ Like a <u>cleaner</u> more <u>direct</u> test just like measurement of angles

There are such tests

1) $B^{\pm} \to DK^{\pm}$ vs $B_s^0 \to D_s^{\mp} K^{\mp}$ → γ ← discrepency ⇒ NP in $B_s^0 - \overline{B_s^0}$ mixing

2) $B_d(t) \to \psi K_s$ vs $B_d(t) \to \phi K_s$ → β ← discrepency ⇒ NP in $b \to s$ penguin

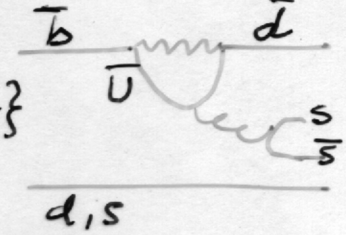
3) $B_s^0(t) \to \psi\phi$ CP asymmetry vanishes, Non zero value ⇒ NP in $B_s^0 - \overline{B_s^0}$ mixing.

All these tests probe NP in $b \to s$ FCNC

Is there any direct test for NP in $b \to d$ FCNC ?

Consider pure $b \to d$ penguin

$$B_d^0 \to K^0 \overline{K^0}$$
$$B_s^0 \to \phi K_S \qquad U = \{u, c, t\}$$



<u>If</u> top quark dominated

$\Rightarrow$ CP asymmetry $a^{cp}_{K^0\overline{K^0}} = 0$ discrepency

$\Rightarrow$ NP in $b \to d$ FCNC

However, $b \to d$ penguin not dominated by internal t

Contribution of u, c as large as 20-50% of t quark

Buras Fliescher
PL 341B, 379(1995)

Are there ways of cleanly measuring the weak phase of t-quark contribution to the $b \to d$ penguin ? No

The general form of the $b \to d$ penguin amplitude

$$P = P_u v_u + P_c v_c + P_t v_t \qquad v_q \equiv V_{qb}^* V_{qd}$$

$V_{ub} \sim e^{-i\gamma}$ $\qquad q = u, c, t$

$V_{td} \sim e^{-i\beta}$ $\qquad v_u + v_c + v_t = 0$ Unitarity relation

Elimating 'u' quark piece

$$P = P_{cu} e^{i\delta_{cu}} + P_{tu} e^{i\delta_{tu}} e^{-i\beta}$$

Imagine that you can cleanly extract β using the above reation i.e. <u>Express '$-\beta$'</u> as a <u>function of observables only</u>.

Now instead of eliminating the 'u' quark contribution, eliminating 't' quark contribution would give

$$P = P_{ct}\, e^{i\delta_{ct}} + P_{ut}\, e^{i\delta_{ut}}\, e^{i\gamma} \quad \text{eliminating '}t\text{'}$$

Earlier we got

$$P = P_{cu}\, e^{i\delta_{cu}} + P_{tu}\, e^{i\delta_{tu}}\, e^{-i\beta} \quad \text{eliminating '}u\text{'}$$

Same method used to extract β can be used to extract γ.

i.e. γ is same function of observables as was used for $-\beta$

$\Rightarrow -\beta = \gamma$ clearly untrue in general

Argument demonstrates that it is <u>impossible</u> to <u>cleanly</u> extract weak phase of top quark in $b \to d$ penguins

This is due to CKM ambiguity

$b \to d$ penguin does not have a well-defined parametrizatio

$b \to s$ penguin

$$P^s = \underbrace{P_u^s V_{ub}^* V_{us}}_{\text{suppressed}} + P_c^s \overbrace{V_{cb}^* V_{cs}} + P_t^s \overbrace{V_{tb}^* V_{ts}}^{\text{real in Wolf par}}$$

Real $\approx$

CKM ambiguity irrelevant in $b \to s$

What does it take to determine the weak phase of t quark in $b \to d$ penguins?

Comparing this phase with the phase of B_d^0-$\overline{B_d^0}$ mixing will test the presence of New Physics (NP)

Let us consider various examples to find out.

We first set up some notation. The time-dependent decay rate for a $B_d^0(t)$ to decay into a final state f is

$$\Gamma(B_d^0(t) \to f) = e^{-\Gamma t} \left[\frac{|A|^2 + |\bar{A}|^2}{2} + \frac{|A|^2 - |\bar{A}|^2}{2} \cos(\Delta M t) - \mathrm{Im}\left(\frac{q}{p} A^* \bar{A}\right) \sin(\Delta M t) \right]$$

Remove the mixing phase by redefining amplitudes. $A \to e^{-i\beta} A, \bar{A} \to e^{-i\beta} \bar{A}$. **Time dependent measurement allows one to extract** $|A|$, $|\bar{A}|$ **and** $\mathrm{Im}(A^* \bar{A})$

1). $B_d^0(t) \to K^0 \overline{K^0}$

$B_d^0 \to K^0 \overline{K^0}$ **is a pure** $b \to d$ **penguin.**

Study of the time-dependent decay rate gives 3 observables $|A|$, $|\bar{A}|$ **and** $\mathrm{Im}(A^* \bar{A})$, **where**

$$\begin{aligned} A &\equiv e^{i\beta} A(B_d^0 \to K^0 \overline{K^0}) \\ \bar{A} &\equiv e^{-i\beta} A(\overline{B_d^0} \to K^0 \overline{K^0}). \end{aligned}$$

$$A = e^{i\delta_{cu}} \left(P_{cu} e^{i\beta} + P_{tu} e^{i(\delta_{tu} - \delta_{cu})} e^{-i\theta_{NP}}\right) .$$

A has 5 unknowns. However β can be independently measured in $B_d^0(t) \to \Psi K_S$.

$$5 - 3 - 1 = 1$$

At least one more unknown than there are measurements.

2). Isospin Analysis of $B \to \pi\pi$

Amplitudes for the decays $B_d^0 \to \pi^+\pi^-$, $B_d^0 \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ form a triangle in isospin space

$$\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0} \qquad 2\ constraints,$$

with a similar triangle relation for the conjugate decays:

$$\frac{1}{\sqrt{2}} \bar{A}^{+-} + \bar{A}^{00} = \bar{A}^{-0} \qquad 2\ constraints.$$

$B_d^0 \to \pi^+\pi^-, \pi^0\pi^0$ receives contributions from a tree diagram and a $b \to d$ penguin diagram. Using unitarity to eliminate the $V_{cb}^* V_{cd}$ piece of the penguin diagram, we can write

$$\frac{1}{\sqrt{2}} A^{+-} = e^{i\delta_P} \left(T^{+-} e^{i(\delta^{+-} - \delta_P)} e^{-i\alpha} + P e^{-i\theta_{NP}} \right) ,$$

$$A^{00} = e^{i\delta_P} \left(T^{00} e^{i(\delta^{00} - \delta_P)} e^{-i\alpha} - P e^{i\delta_P} e^{-i\theta_{NP}} \right) ,$$

$$A^{+0} = e^{i\delta_P} \left(T^{+-} e^{i(\delta^{+-} - \delta_P)} + T^{00} e^{i(\delta^{00} - \delta_P)} \right) e^{-i\alpha} .$$

T's include u quark piece of the penguin amplitude. The magnitudes of the six amplitudes $|A^{+-}|$, $|A^{00}|$, $|A^{+0}|$, $|\bar{A}^{+-}|$, $|\bar{A}^{00}|$ and $|\bar{A}^{-0}|$, can be measured experimentally. In addition one can measure 5 relative phases between these quantities.

There is however one further constraint $|A^{+0}| = |\bar{A}^{-0}|$.

$$7 - (11 - 5) = 1$$

We still have one more unknown than there are measurements.

3). Dalitz Plot Analysis for $B \to 3\pi$

Here $B \to \rho\pi$ amplitudes are the key ingredients. Isospin allows one to relate neutral $B \to \rho\pi$ decays to charged $B \to \rho\pi$ decays. Defining

$$\begin{aligned}
S_1 &\equiv e^{i\beta}\sqrt{2}A(B^+ \to \rho^+\pi^0) \ , \\
S_2 &\equiv e^{i\beta}\sqrt{2}A(B^+ \to \rho^0\pi^+) \ , \\
S_3 &\equiv e^{i\beta}A(B_d^0 \to \rho^+\pi^-) \ , \\
S_4 &\equiv e^{i\beta}A(B_d^0 \to \rho^-\pi^+) \ , \\
S_5 &\equiv e^{i\beta}2A(B_d^0 \to \rho^0\pi^0) \ ,
\end{aligned}$$

Eliminating the $V_{cb}^* V_{cd}$ piece, the above amplitudes can be written explicitly as follows :

$$\begin{aligned}
S_1 &= T^{+0}e^{i\delta^{+0}}e^{-i\alpha} + 2P_1 e^{i\delta_1}e^{-i\theta_{NP}} \ , \\
S_2 &= T^{0+}e^{i\delta^{0+}}e^{-i\alpha} - 2P_1 e^{i\delta_1}e^{-i\theta_{NP}} \ , \\
S_3 &= T^{+-}e^{i\delta^{+-}}e^{-i\alpha} + P_1 e^{i\delta_1}e^{-i\theta_{NP}} + P_0 e^{i\delta_0}e^{-i\theta_{NP}} \ , \\
S_4 &= T^{-+}e^{i\delta^{-+}}e^{-i\alpha} - P_1 e^{i\delta_1}e^{-i\theta_{NP}} + P_0 e^{i\delta_0}e^{-i\theta_{NP}} \ , \\
S_5 &= -T^{+-}e^{i\delta^{+-}}e^{-i\alpha} - T^{-+}e^{i\delta^{-+}}e^{-i\alpha} + T^{+0}e^{i\delta^{+0}}e^{-i\alpha} \\
&\quad + T^{0+}e^{i\delta^{0+}}e^{-i\alpha} - 2P_0 e^{i\delta_0}e^{-i\theta_{NP}} \ .
\end{aligned}$$

The Dalitz plot of the $\pi^+\pi^-\pi^0$ final state contains enough information to determine the magnitudes and relative phases of the six amplitudes S_3, S_4, S_5, $\bar{S}_3$, $\bar{S}_4$ and $\bar{S}_5$. S_1, S_2, $\bar{S}_1$ and $\bar{S}_2$ can be obtained from an analysis of the Dalitz plot of $\pi^+\pi^0\pi^0$. Thus, there are nominally 19 measurements, 10 amplitudes 9 phases.

There are however constraints:

- **Isospin pentagon relations,**

$$S_1 + S_2 = S_3 + S_4 + S_5 \qquad 2\ constraints,$$

$$\bar{S}_1 + \bar{S}_2 = \bar{S}_3 + \bar{S}_4 + \bar{S}_5 \qquad 2\ constraints,$$

  mean S_5 and $\bar{S}_5$ are not independent. This removes 4 measurements.

- **We have the equality $|S_1 + S_2| = |\bar{S}_1 + \bar{S}_2|$. This removes one more measurement.**

- **It is easy to verify the complex equality**

$$\frac{S_3 - S_4 - S_1}{\bar{S}_3 - \bar{S}_4 - \bar{S}_1} = \frac{S_1 + S_2}{\bar{S}_1 + \bar{S}_2}\,.$$

  This removes 2 more measurements.

$$13 - (19 - 7) = 1$$

We still have one more unknown than there are measurements.

4). Angular Analysis of $B \to VV$ Decays

The fundamental quantities in all observables are the six amplitudes A_λ and $\bar{A}_\lambda$, $\lambda = 0, \perp, \|$. The most one can measure is their magnitudes and relative phases, for a total of 11 independent measurements.

The total number of theoretical parameters in the decay amplitudes are 13: β, θ_{NP}, 6 magnitudes of amplitudes – 2 for each of the 3 helicity, and 5 relative strong phases. Assuming that β is independently measured reduces one unknown.

$$13 - 1 - 11 = 1$$

We still have one more unknown than there are measurements.

Conclusion:

It is possible to obtain the weak phase of the penguin contributions, if one makes a single assumption involving the hadronic parameters. With such an assumption, one can test for the presence of new physics in the $b \to d$ flavour-changing neutral current by comparing the weak phase of B_d^0-$\overline{B_d^0}$ mixing with that of the t-quark contribution to the $b \to d$ penguin.